Upper and lower fatigue life limits model using energy-based fatigue properties

H. Jahed a,1, A. Varvani-Farahani b,*

a Department of Mechanical Engineering, Iran University of Science and Technology (IUST), Tehran 16844, Iran
b Department of Mechanical and Industrial Engineering, Ryerson University, 350 Victoria Street, Toronto, Ont., Canada M5B 2K3

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Abstract

Cyclic strain-life data and corresponding Coffin–Manson coefficients for both normal and shear strain-lives were first defined. Energy–fatigue life curves were then generated from strain-fatigue life properties. The upper and lower limits of life are estimated using the proposed life equations. The upper life limit is obtained by assuming that the dominant cracking mechanism is Case A and the lower life limit is obtained by assuming that the dominant cracking mechanism is Case B. The proposed method was developed based on physical evidences of crack initiation and growth as well as the amount of dissipated energy over life cycles. The fatigue life data fall between the upper and the lower limits resulting in a promising life prediction. The proposed method has been used to evaluate the fatigue life of various metallic materials of SAE 1045, AISI 304, Inc 718 and Haynes 188 reported in the literature. Results of fatigue life predictions were found in good agreement with experimental life data.

Keywords: Low cycle fatigue; Life limits; Energy–life curves; Life prediction model; Fatigue properties

1. Introduction

In spite of decades of investigation, fatigue response of materials is yet to be fully understood. This is partially due to the complexity of loading at which two or more loading axes fluctuate with time. Examples of structures experiencing such complex loadings are automobile, aircraft, off-shores, railways and nuclear plants. Fluctuations of stress and/or strains are difficult to avoid in many practical engineering situations and are very important in design against fatigue failure.

A comprehensive review on multiaxial fatigue reported over last two decades distinguishes two most promising fatigue damage criteria: (i) criteria based on applied cyclic strain/stress components, and (ii) criteria based on tensorial strain and stress components. Critical plane approaches are in the first group and the energy-based methods fall in the second category. A full review of these approaches has been addressed by Brown and Miller [1], Krempl [2], Garud [3], Bannantine [4], Socie [5], Macha and Sonsino [6], and Carpinteri and Spagnoli [7].

Though the importance of maximum shear plane goes back to early studies of fatigue, it was not until Findley [8] who put it into formulation. He proposed the addition of alternating shear stress and normal stress component on the plane of maximum shear as the controlling parameter of fatigue. Forsyth [9] defined fatigue crack growth as a two-stage process, where stage I corresponds to Mode II crack growth (caused by shear component) and stage II corresponds to Mode I crack growth (induced by normal components). This justified the use of both shear and normal components in fatigue parameter definition. Later, McDiarmid [10] modified the Findley’s stress-based model to consider different cracking responses. Since the plasticity involved in fatigue is best observed by strain formulation, Brown and Miller [11] proposed the strain version of the Findley’s parameter. Fatemi and Socie [12] further modified the plane of maximum shear strain by incorporating a stress component and thus taking into account additional hardening for out-of-phase loading conditions [13].

In 1987, Socie [14] rearranged Smith–Watson–Topper (SWT) [15] parameter to define the critical plane based on the crack initiation plane and the amount of energy dissipated during fatigue loading. Chu et al. [16] used both shear and normal components on critical planes to define a parameter
similar to an energy term. Liu [17] proposed a virtual energy method that could be used for materials with different cracking mechanisms. As the latest improvement, Varvani-Farahani [18] proposed an energy-based non-dimensional parameter holding terms for material properties, mean stress effect and additional hardening for out-of-phase loading conditions.

Energy concept for fatigue damage assessment of engineering materials has been long discussed since 1927 [19]. Feltner and Morrow [20] employed cyclic stress–strain hysteresis loops as an energy concept in fatigue analysis. Their work was mainly based on experimental results. Cyclic plasticity models of Mroz [21] and Garud [22] in conjunction with suitable hardening rules have always been required to assess fatigue damage under low cycle fatigue. Such cyclic plasticity theories are used to calculate stress and strain components. Due to strain path dependency and yield surface evolution, stress/strain calculation is complicated and requires rather complex mathematical expressions particularly for out-of-phase loading conditions.

The present study aims to address an energy-based parameter to evaluate the fatigue life of metallic components subjected to multiaxial loading conditions. The proposed parameter predicts the fatigue lives with a good degree of success as compared with experimental values of life for various metallic materials (SAE 1045, AISI 304, Inconel 718, and Haynes 188 reported in the literature).

### 2. Proposed fatigue damage assessment

#### 2.1. Strain-fatigue life data and Coffin–Manson equations

In 1900, Basquin showed that the relationship between stress and life could be linearized by employing full log–log coordinates, and thereby established the exponential law of fatigue. Fifty-five years later, Coffin and Manson established that the plastic strain-life data could also be linearized with log–log coordinates. The equations of Basquin and Coffin–Manson hold the stress-based and strain-based fatigue properties, respectively. The summation of these equations is known as Coffin–Manson equation:

\[
\frac{\Delta \varepsilon}{2} = \frac{\sigma' i}{E} (2N_t)^b + \varepsilon'_t (2N_t)^c
\]  

(1)

where \( \sigma' i \) and \( b \) are fatigue strength coefficient and fatigue strength exponent, respectively, and the \( \varepsilon'_t \) and \( c \) correspond to fatigue ductility coefficient and fatigue ductility exponent, respectively. Number of cycles to failure is denoted by \( N_t \). An equation similar to Eq. (1) is used to express shear strain-fatigue life data as:

\[
\frac{\Delta \gamma}{2} = \frac{\tau'_t}{G} (2N_t)^b + \gamma'_t (2N_t)^c
\]  

(2)

where \( \tau'_t \) and \( b \) are shear fatigue strength coefficient and shear fatigue strength exponent, respectively, and the \( \gamma'_t \) and \( c \) correspond to shear fatigue ductility coefficient and shear fatigue ductility exponent, respectively.

#### 2.2. Energy-fatigue life data and Coffin–Manson type equations

Similar to Eqs. (1) and (2) which are used for strain-based approaches, it is logical that for an energy approach the corresponding \( \Delta E–N \) (energy–life) curve be used. Moreover, the fatigue life prediction equation should employ energy-based fatigue properties.

Initially, the strain-life coefficients were determined using Coffin–Manson equations (Eqs. (1) and (2)). Energy–fatigue life curves were then constructed using strain-based fatigue properties discussed in Section 2.1. This is done by calculating the strain energy at each point of curves provided by Coffin–Manson equations and relating them to the corresponding life. Energy-based fatigue properties are then defined based on log plastic energy-log life and log elastic energy-log life of the tensile and torsion energy–life curves, respectively. Two different lives are then estimated using two life prediction equations, similar to Coffin–Manson equation, which

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( B )</td>
<td>axial energy-based fatigue strength exponent</td>
</tr>
<tr>
<td>( b )</td>
<td>fatigue strength exponent</td>
</tr>
<tr>
<td>( B_s )</td>
<td>shear energy-based fatigue strength exponent</td>
</tr>
<tr>
<td>( b_s )</td>
<td>shear fatigue strength exponent</td>
</tr>
<tr>
<td>( C )</td>
<td>axial fatigue toughness exponent</td>
</tr>
<tr>
<td>( c )</td>
<td>fatigue ductility exponent</td>
</tr>
<tr>
<td>( C_s )</td>
<td>shear fatigue toughness exponent</td>
</tr>
<tr>
<td>( c_s )</td>
<td>shear fatigue ductility exponent</td>
</tr>
<tr>
<td>( E )</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>( E'_f )</td>
<td>axial fatigue strength coefficient</td>
</tr>
<tr>
<td>( E'_t )</td>
<td>axial fatigue toughness</td>
</tr>
<tr>
<td>( G )</td>
<td>shear modulus</td>
</tr>
<tr>
<td>( N_A )</td>
<td>fatigue life in a purely axial loading</td>
</tr>
<tr>
<td>( N_f )</td>
<td>fatigue life</td>
</tr>
<tr>
<td>( N_T )</td>
<td>fatigue life in a purely torsional loading</td>
</tr>
<tr>
<td>( W'_c )</td>
<td>shear fatigue strength coefficient</td>
</tr>
<tr>
<td>( W'_f )</td>
<td>torsional fatigue toughness</td>
</tr>
<tr>
<td>( \Delta E )</td>
<td>strain energy density range</td>
</tr>
<tr>
<td>( \Delta E_A )</td>
<td>axial strain energy density range</td>
</tr>
<tr>
<td>( \Delta E_T )</td>
<td>shear strain energy density range</td>
</tr>
<tr>
<td>( \Delta \varepsilon )</td>
<td>strain range</td>
</tr>
<tr>
<td>( \Delta \sigma )</td>
<td>stress range</td>
</tr>
<tr>
<td>( \varepsilon'_t )</td>
<td>fatigue ductility coefficient</td>
</tr>
<tr>
<td>( \gamma'_t )</td>
<td>shear fatigue ductility coefficient</td>
</tr>
<tr>
<td>( \sigma'_t )</td>
<td>fatigue strength coefficient</td>
</tr>
<tr>
<td>( \tau'_t )</td>
<td>shear fatigue strength coefficient</td>
</tr>
</tbody>
</table>

### Additional Values

- \( f \): Fatigue ductility coefficient
- \( c_0 \): Fatigue ductility exponent
- \( f_0 \): Shear fatigue strength coefficient
- \( s_0 \): Shear fatigue strength exponent
- \( E_y \): Tensile yield strength
- \( E_u \): Ultimate tensile strength
- \( E_0 \): Elastic modulus
- \( N_f \): Fatigue life
- \( N_T \): Fatigue life
- \( W'_c \): Shear fatigue strength coefficient
- \( W'_f \): Torsional fatigue toughness
- \( \Delta E \): Strain energy density range
- \( \Delta E_A \): Axial strain energy density range
- \( \Delta E_T \): Shear strain energy density range
- \( \Delta \varepsilon \): Strain range
- \( \Delta \sigma \): Stress range
- \( \varepsilon'_t \): Fatigue ductility coefficient
- \( \gamma'_t \): Shear fatigue ductility coefficient
- \( \sigma'_t \): Fatigue strength coefficient
- \( \tau'_t \): Shear fatigue strength coefficient

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This summary covers the key concepts and equations related to fatigue damage assessment, focusing on the Coffin–Manson model and its applications to various material properties and loading conditions. The text underscores the importance of understanding the exponential law of fatigue and its linearization through log–log coordinates. It also highlights the use of energy concepts to evaluate fatigue damage, particularly in the context of metallic components under multiaxial loading.
incorporate new sets of fatigue properties:

\[
\Delta E_A = E'_e(N_A)^B + E'_f(N_A)^C \tag{3}
\]

\[
\Delta E_T = W'_e(N_T)^B + W'_f(N_T)^C \tag{4}
\]

where \(\Delta E_A\) and \(\Delta E_T\) are the energy due to a purely tensile loading and the energy due to a purely torsion loading, respectively. Coefficients \(E'_e\) and \(W'_e\) are the axial and torsion fatigue toughness, respectively \([23]\). Terms \(E'_e\) and \(W'_e\) correspond to the axial and the shear fatigue strength coefficients, respectively. Exponents \(C\), \(C_s\) and \(B\), \(B_s\) are the axial and shear fatigue toughness exponents and axial and shear energy-based fatigue strength exponents, respectively. In Eqs (3) and (4), \(N_A\) and \(N_T\) are lives obtained in purely axial and torsional loading conditions, respectively.

The plot of energy versus fatigue life (\(\Delta E-N\)) is shown in Fig. 1.

Coefficients \(E'_e\) and \(W'_e\) were obtained from the intersects of the plastic energy–life curves at the first cycle and \(C\), \(C_s\) are the corresponding slopes in the axial and torsion energy curves, respectively. Similarly, coefficients \(E'_f\) and \(W'_f\) were obtained from the intersects of the elastic energy–life curves at the first cycle and \(B\), \(B_s\) are the corresponding slopes in the axial and torsion energy curves, respectively. The calculated values for AISI 304, Inc 718 and SAE 1045 found in similar way using data from Refs. \([14,24,25–27]\) are tabulated in Table 1 and compared with commonly used coefficients in the modified Coffin–Manson equation. The percentage difference of the defined energy-based parameter and commonly used coefficients are also included in Table 1.

Table 1

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Axial</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma^2/E) (MJ/m³)</td>
<td>4.685</td>
<td>-22.97</td>
<td>5.405</td>
<td>-18.01</td>
<td>75</td>
<td>-1204.35</td>
</tr>
<tr>
<td>(E'_e) (MJ/m³)</td>
<td>3.81</td>
<td>4.58</td>
<td>34.53</td>
<td>5925</td>
<td>5.75</td>
<td>6.03</td>
</tr>
<tr>
<td>(\sigma \times \varepsilon'_e) (MJ/m³)</td>
<td>196</td>
<td>51.72</td>
<td>171</td>
<td>34.53</td>
<td>5925</td>
<td>5.75</td>
</tr>
<tr>
<td>(E'_f) (MJ/m³)</td>
<td>406</td>
<td>261.2</td>
<td>171</td>
<td>34.53</td>
<td>5925</td>
<td>5.75</td>
</tr>
<tr>
<td>(c+b)</td>
<td>-0.54</td>
<td>3.05</td>
<td>-0.516</td>
<td>-0.19</td>
<td>-0.912</td>
<td>-12.23</td>
</tr>
<tr>
<td>(C)</td>
<td>-0.557</td>
<td>-0.515</td>
<td>-0.19</td>
<td>-0.912</td>
<td>-0.8126</td>
<td>-12.23</td>
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<tr>
<td>(2 \times b)</td>
<td>-0.22</td>
<td>-14.58</td>
<td>-0.228</td>
<td>-0.87</td>
<td>-0.302</td>
<td>-202.00</td>
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<tr>
<td>(B)</td>
<td>-0.192</td>
<td>-0.23</td>
<td>-0.23</td>
<td>-0.1</td>
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<tr>
<td>Torsion</td>
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<td></td>
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<tr>
<td>(\tau^2/G) (MJ/m³)</td>
<td>3.23</td>
<td>4.15</td>
<td>6.071</td>
<td>37.04</td>
<td>59.2</td>
<td>-798.33</td>
</tr>
<tr>
<td>(W'_e) (MJ/m³)</td>
<td>3.37</td>
<td>4.43</td>
<td>6.071</td>
<td>37.04</td>
<td>59.2</td>
<td>-798.33</td>
</tr>
<tr>
<td>(\tau \times \gamma'_f) (MJ/m³)</td>
<td>208.6</td>
<td>54.39</td>
<td>292.82</td>
<td>37.70</td>
<td>38628</td>
<td>-17.29</td>
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<tr>
<td>(W'_f) (MJ/m³)</td>
<td>457.4</td>
<td>470</td>
<td>32933</td>
<td>38628</td>
<td></td>
<td></td>
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<tr>
<td>(c+b_s)</td>
<td>-0.5594</td>
<td>0.59</td>
<td>-0.474</td>
<td>6.69</td>
<td>-1.07</td>
<td>-9.74</td>
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<tr>
<td>(C_s)</td>
<td>-0.5627</td>
<td>-0.508</td>
<td>-0.474</td>
<td>6.69</td>
<td>-1.07</td>
<td>-9.74</td>
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<tr>
<td>(2 \times b_s)</td>
<td>-0.2108</td>
<td>-7.50</td>
<td>-0.242</td>
<td>-6.84</td>
<td>-0.296</td>
<td>-138.13</td>
</tr>
<tr>
<td>(B_s)</td>
<td>-0.1961</td>
<td>-0.2265</td>
<td>-0.242</td>
<td>-6.84</td>
<td>-0.296</td>
<td>-138.13</td>
</tr>
</tbody>
</table>
2.3. Fatigue life prediction model

The primary intention of this study is to develop a fatigue model to accurately predict the life of engineering components. The proposed parameter is able to address fatigue life assessment of metallic materials through a number of steps as:

(i) Determine the energy-based fatigue coefficients/properties of materials.

(ii) Calculate cyclic energy for a given component knowing material, geometry and cyclic loading. This is done by performing elastic–plastic analysis using cyclic behavior of material. Cyclic energy will be calculated by integrating both elastic and plastic components of energy from cyclic hysteresis loops. To calculate energy terms, both stress and strain components are required. In one-dimensional cases, Ramberg-Osgood equation is employed to calculate stress terms when strain values are known (and vice versa).

(iii) Calculate two values of fatigue life while equating left-hand sides of Eqs. (3) and (4) with total energy value determined from step (ii). These equations are now used by assuming that the total energy in the engineering component is purely axial and torsional, respectively. At this stage, two fatigue life values are determined which correspond to the upper and the lower fatigue life limits.

For a component experiencing Case A cracking [11] where shear cracks growing along the surface, crack growth is very slow while the life of such component under Case B cracking where shear cracks growing into the surface is much smaller. The upper and the lower limits of fatigue life are logically coincided with these two proposed life prediction models. If the cracking response in the material is found purely in the coincided with these two proposed life prediction models. If the cracking response in the material is found purely in the

components subjected to non-proportional loading if the cyclic energies are provided. Further investigation is planned by the present authors to develop the method to be applied for non-proportional loading conditions. Nitta et al. [28] have developed an energy-based model to account the non-proportionality effect contributing both axial and torsional (shear) strain energy to predict fatigue life. In their work, extensive experimental data were needed to estimate axial and shear lives.

3. Evaluation of the proposed method

To further evaluate the proposed energy-based fatigue parameter, several sets of fatigue data for various materials reported in the literature [14,24–27] have been studied.

Tables 1 and 2 lists the fatigue coefficients and their corresponding energy-based fatigue properties for the following engineering materials.

3.1. SAE 1045 steels

The data include simple tension, torsion and proportional tension–torsion tests with various strain ratios [25–27]. It is
interesting to note that the energy-based properties in tension and torsion are very similar, thus expecting similar life in uniaxial loading and torsion. Fig. 2 presents the results of the upper and lower life limits for 1045 steel. Both life limits are very similar and close to the experimental fatigue lives. Fig. 3 shows how successfully the proposed damage model can correlate the experimental and the predicted fatigue lives within a factor of $\pm 1.8$ under various loading conditions.

3.2. Inconel 718

Inconel 718 components were tested under axial and torsional fatigue loading. The cyclic properties of this material have been reported in reference [24]. The test results include loading in pure tension (with loading ratio $\lambda=0$) with and without mean strain, pure torsion ($\lambda=\infty$) with and without mean strain, and in-phase tension–torsion ($\lambda=1.7$ and $\lambda=1.8$) with and without mean strain. Fig. 4 shows that the experimental tension–torsion fatigue data fall between predicted lower and upper life limits $N_A$ and $N_T$, based on the parameter proposed in this study. It can be observed that the experimental life is lower than $N_T$ and higher than $N_A$. Experimental fatigue lives obtained under uniaxial, torsional and tension–torsion loading conditions are compared with the predicted lives determined by the present method, see Fig. 5. This figure shows very good agreement of predicted lives as plotted versus the experimental lives. The agreement factor varies between $\pm 1.2$.

3.3. AISI 304 steel

Fatigue tests were performed under axial and torsional loading conditions. The loading path, testing conditions and cyclic properties are available in reference [14]. The test results...
for loading path A (with loading ratio $\lambda = 0$), path B ($\lambda = \infty$) and path C ($\lambda = 1.7$ and $\lambda = 1.6$) have been examined. Fig. 6 shows that the experimental data fall between the predicted upper and the lower life limits. Predicted life obtained by employing the present parameter is also compared to the experimental life data in Fig. 7 and is found within promising life factor range equal to $\pm 1.3$.

3.4. Haynes 188 at 760 °C

Fatigue tests under axial and torsional loading were performed on this material [24] loaded at 760 °C. The corresponding axial and torsional fatigue properties are given in Table 2. Fatigue test results include only in-phase tension–torsion loading with the strains ratios of $\lambda = 0.87$, 3.48 and 1.7. Fig. 8 shows that the experimental fatigue data are between the lower and the upper life limits $N_A$ and $N_T$. It can also be observed that the experimental life data fall under $N_T$-line and above $N_A$-line. Predicted fatigue lives obtained using the present method are plotted versus the experimental life in Fig. 9. This figure shows an agreement factor in the range of $\pm 1.3$ between experimental and predicted lives even at an elevated temperature of 760 °C.

4. Conclusions

An energy-based fatigue approach has been herein proposed to assess the fatigue lives of engineering components. This method is based on the cracking mechanisms and the amount of dissipated energy during fatigue loading cycles. It is shown that the experimentally obtained fatigue lives fall within the predicted upper and lower bounds. The results of fatigue life predictions based on the proposed parameter show a good agreement between predicted and experimental lives for all sets of materials tested under axial, torsional and tension–torsion fatigue loading conditions. The proposed method correlates the predicted and experimental fatigue lives within factors of $G = 1.8$, $G = 1.2$, $G = 1.3$, and $G = 1.3$ for SAE 1045 steel, Inconel 718, AISI 304 steel, and Haynes 188, respectively.

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